



Optimal Control Design of Active Suspension System Based on Quarter Car Model

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Abstract — The optimal control design of the ground-vehicle active suspension system is presented. The active suspension system is to improve the vehicle ride comfort by isolating vibrations induced by the road profile and vehicle velocity. The vehicle suspension system is approached by a quarter car model. Dynamic equations of the system are derived by applying Newton's second law. The control law of the active suspension system is designed using linear quadratic regulator (LQR) method. Performance evaluation is done by benchmarking the active suspension system to a passive suspension system. Both suspension systems are simulated in computer. The simulation results show that the active suspension system significantly improves the vehicle ride comfort of the passive suspension system by reducing 50.37% RMS of vertical displacement, 45.29% RMS of vertical velocity, and 1.77% RMS of vertical acceleration.

Keywords – Optimal control, linear quadratic regulator, active suspension system, ride comfort, quarter car.

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I. INTRODUCTION

One of the purposes of applying the suspension system on a ground vehicle is to improve vehicle ride comfort [1]. The ride comfort is essential in modern vehicles. A suspension system works by isolating the vehicle from vibration induced by the vehicle velocity and road profile. Vehicle suspension systems are classified into three types, i.e., passive suspension system, semi-active suspension system, and active suspension system [1–5]. The passive suspension system is the conventional suspension system that consists of spring and damper. Both spring and damper have a fixed spring constant and fixed damping rate, respectively. A semi-active suspension system is similar to the passive suspension system, but damping rate of the damper is varying. The damping rate is high for low velocities but low for high velocities. An active suspension is another type of suspension system, where an active element is added to the spring and damper. The active element is an actuator to generate force for suppressing the vibration. The actuator works based on control commands generated by a controller. The controller calculated the control commands based on a state feedback control law to calculate the control command. Studies on suspension systems show that

active suspension system resulted in the best performance followed by the semi-active suspension system and the passive suspension system [2–10].

The active suspension system becomes an interesting research topic due to the superiority of the suspension performance. Several research works on developing an active suspension system have been presented in the last five years [2, 7–9, 11–15]. The active suspension systems were developed by the following steps: modeling the vehicle suspension system dynamics, designing a state feedback control for the active suspension system, and evaluating the active suspension system performance through simulation and/or experimental tests. The modeling was done by applying physical laws. There are three models of vehicle suspension system: quarter-car model, half-car model, and full-car model. The quarter-car model is a one-dimensional (1D) model that only captures the heaving motion of the vehicle [2, 6, 15]. The half-car model is a two-dimensional (2D) model that can capture the heaving and pitching motions [16]. The full-car model is a three-dimensional (3D) model that captures the heaving, pitching, and rolling motions of the vehicle [7]. The vehicle suspension system models divide the vehicle into two types of mass: sprung mass and unsprung

mass. The sprung mass is a mass of vehicle parts that are supported by the suspension system, for example body, frame, engine, interior, passengers, and cargo. The unsprung mass is mass of the vehicle parts that are not supported by the suspension system but by the tires, for example, wheels, tires, brake assemblies, and wheel shafts. Two masses of vehicles increase degree of freedom (DoF) of the model. The quarter car model is, therefore, to have two degrees of freedom (2-DoF), the half-car model to have 4-DoF, and the full car model to have 6-DoF.

The higher DoF of suspension system gives a challenge in developing the controller of suspension systems. The suspension systems are either single-input multi-output (SIMO) systems or multi-input multi-output (MIMO) systems. Therefore, dynamic equations of the vehicle suspension system are appropriately presented in the state-space form. Most of the studies on active suspension system applied multivariable control methods in developing the controller, for examples: linear quadratic regulator (LQR) [6, 17-19], linear quadratic Gaussian (LQG) [20-22], optimal preview control [7, 11, 23], H-infinity [12, 15], sliding mode control [9, 14, 24], and predictive control [8]. The studies show that the optimal control methods were intensively applied in the active suspension system development. The optimal control methods are including LQR, LQG, and optimal preview control.

This paper presents a comprehensive study on developing an active suspension system using a linear quadratic regulator (LQR) method based on a quarter car model. The study includes modeling the suspension system, tuning the LQR cost function matrices, determining the suspension system characteristic, and evaluating the suspension system performance. The paper is organized as follows. The introduction is presented in Section I. Section II describes the method in developing the active suspension system, including modeling the active suspension system, state feedback control, and control design using LQR. Section III provides results of simulation where performance of the designed active suspension system performance is compared to performance of a passive suspension system. Discussion on the developing active suspension system and results are presented in Section IV. Finally, this study is concluded in Section V.

II. RESEARCH METHOD

A. Suspension System Dynamics

Figure 1 shows a model of a quarter car suspension system. The model has two masses: m_1 and m_2 , where the m_1 is the unsprung mass and the m_2 is the sprung mass. The suspension consists of three elements: a spring with a stiffness coefficient k_2 , a damper with damping coefficient c , and an actuator that generates force u . Tire of the car is modeled as a spring with stiffness k_1 .

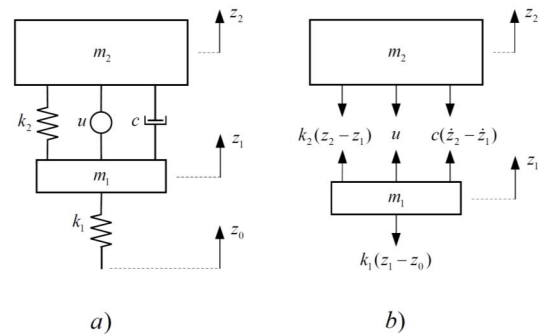


Fig. 1. a) A Model of Quarter-Car Suspension System. b) Free Body Diagram of The Quarter-Car Suspension System.

Dynamic equations of the suspension system are derived based on Newton's second law. Newton's second law states that forces working on an object are expressed by (1),

$$\Sigma F = ma \tag{1}$$

Where F is the resultant forces working on the object, m is mass of the object, and a is an acceleration of the object. The quarter-car suspension system dynamics are derived as follows. Dynamic equation of the unsprung mass is given by (2),

$$k_2(z_2 - z_1) + u + c(\dot{z}_2 - \dot{z}_1) - k_1(z_1 - z_0) = m_1\ddot{z}_1 \tag{2}$$

and rearranging it results in:

$$\ddot{z}_1 = \frac{c}{m_1}(\dot{z}_2 - \dot{z}_1) - \frac{k_1+k_2}{m_1}z_1 + \frac{k_2}{m_1}z_2 + \frac{1}{m_1}u + \frac{k_1}{m_1}z_0. \tag{3}$$

While the dynamics equation of the sprung mass is given by (4),

$$-k_2(z_2 - z_1) - u - c(\dot{z}_2 - \dot{z}_1) = m_2\ddot{z}_2 \tag{4}$$

and rearrange it results in:

$$\ddot{z}_2 = -\frac{c}{m_2}(\dot{z}_2 - \dot{z}_1) - \frac{k_2}{m_2}(z_2 - z_1) - \frac{u}{m_2}. \tag{5}$$

The equations (3) and (5) are explicitly described the quarter car dynamic equations. For the (3) and (5), define the following system states,

$$\begin{aligned} x_1 &= z_1, & x_2 &= z_2, & x_3 &= \dot{z}_1, \\ x_4 &= \dot{z}_2, & w &= z_0. \end{aligned} \tag{6}$$

Using the defined states, dynamics equations of the quarter car suspension system (4) and (5) can be expressed in a state-space form as follows,

$$\dot{x} = Ax + Bu + Dw \tag{7}$$

where the vector x and matrices A , B , and D are defined as follows,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_1+k_2}{m_1} & -\frac{c}{m_1} & \frac{k_2}{m_1} & \frac{c}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c}{m_2} & -\frac{k_2}{m_2} & -\frac{c}{m_2} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{-1}{m_2} \end{bmatrix}, \text{ and } D = \begin{bmatrix} 0 \\ \frac{k_1}{m_1} \\ 0 \\ 0 \end{bmatrix}. \quad (8)$$

The notation x is the system states vector, A is the system matrix, B is the input matrix, u is the system input, D is the disturbance matrix, and w is the system disturbance.

B. States Feedback Control

Defining the system input u in (7) equals to the following state feedback control,

$$u = -Kx \quad (9)$$

Where K is the control gain matrix. Substituting (9) into (7) results in,

$$\dot{x} = (A - BK)x + Dw \quad (10)$$

Which is known as a closed-loop system equation of the quarter-car suspension system. For the closed-loop system, defining

$$A_c = A - BK \quad (11)$$

where A_c is the closed-loop system matrix and substituting it into (10) results in,

$$\dot{x} = A_c x + Dw \quad (12)$$

The state feedback control law (9) is designed to obtain a control gain matrices K such that closed-loop system matrix A_c is Hurwitz [25, 26]. The next subsection will discuss a control design using a linear quadratic regulator (LQR) method to find the K .

C. Linear Quadratic Regulator

Linear quadratic regulator (LQR) is a part of optimal control [27]. The optimal control provides a control design method for a multivariable system, where the control gain matrix is determined through minimizing a cost function. The LQR is specified by a linear quadratic cost function, and the system states are assumed to be available. The control design of the active suspension system, (7) using LQR is explained as follows.

Define the following cost function for (7),

$$J = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (13)$$

Where J is the cost function, Q is a symmetric positive semi-definite matrix, and R is a positive definite matrix. The optimal control problem is to find u such that the J is minimized. Solution for the optimal control problem is a state feedback control given as follows [27],

$$u = -Kx \quad (14)$$

Where K is the control gain matrix. Through a calculation based on unconstrained optimization, it has resulted:

$$K = R^{-1} B^T P \quad (15)$$

Where P is a symmetric matrix. The (15) shows that the control gain K is determined by the matrices R , B , and P . The matrix P is obtained through solving the following algebraic Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \quad (16)$$

The (16) has one unknown matrix, which is P , and four known matrices, which are A , B , Q , and R . The matrices A and B are the known matrices given by the suspension system dynamics, while the matrices Q and R are defined in the control design process.

The Q and R are known as the control design parameters of the LQR method. Since the system (7) has four states and one input, the matrix Q is a four-by-four matrix, and the matrix R is a one-by-one matrix or a scalar. Define Q as a diagonal matrix given as follows:

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix} \quad (17)$$

has resulted substituting it into (13) results in:

$$J = \frac{1}{2} \int_0^{\infty} (q_1 x_1^2 + q_2 x_2^2 + q_3 x_3^2 + q_4 x_4^2 + u^T R u) dt \quad (18)$$

The (18) shows that the diagonal elements of matrix Q and the scalar R become the weighting factor for the system states and the control input in the cost function, respectively.

III. RESULT

The performance of the designed active suspension system is evaluated through computer simulation. The simulation diagram is shown in Fig.2. The performance evaluation is done by benchmarking the active suspension system performance to a passive suspension system performance. Therefore, two simulation programs are created. One simulation program is to simulate a vehicle equipped with the active suspension system and another simulation program is to simulate a similar vehicle but equipped with the passive suspension system. Parameters of the vehicle, the active suspension, and the passive suspension are given in Table I.

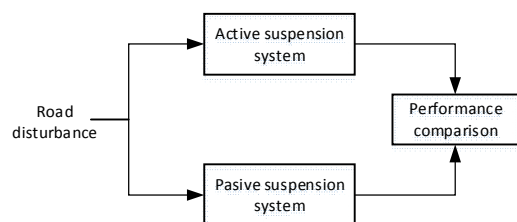


Fig.2. Diagram of simulating the active and passive suspension system

Table 1. Vehicle and Suspension Parameters [6]

Parameter	Symbol	Value	Unit
Vehicle			
Unsprung mass	m_1	28.58	kg
Sprung mass	m_2	288.9	kg
Tyre stiffness	k_1	1.559×10^5	N/s
Active Suspension			
Spring stiffness	k_2	1×10^5	N/s
Damping coefficient	c	850	Ns/m
Passive Suspension			
Spring stiffness	k_2	19.96×10^3	N/s
Damping coefficient	c	1300	Ns/m

The active suspension system is designed using the LQR method, where the control law is given in (14). The control gain matrix of (14) is designed by selecting the matrices Q and R , and the following matrices are selected in this study:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10^5 & 0 & 0 \\ 0 & 0 & 10^3 & 0 \\ 0 & 0 & 0 & 10^8 \end{bmatrix} \text{ and } R = 1. \quad (19)$$

The matrix elements $Q(1,1)$ and $Q(2,2)$ are the weighting factor of x_1 and x_2 . The value of $Q(2,2)$ higher than $Q(1,1)$ indicates that the x_2 is desired to be more minimized than x_1 . Similarly for $Q(3,3)$ and $Q(4,4)$ where the x_4 is desired to be more minimized than the x_3 .

Applying the active suspension system (7) with the system parameters given in Table I and the cost function matrices Q and R defined in (17) into the LQR algorithm results in the following control gain matrix:

$$K = [11700 \quad -224.6 \quad 5 \times 10^{-3} \quad -9400] \quad (20)$$

Substituting the resulted K into (11) yields in the closed-loop system matrix where the active suspension system characteristic is determined by eigenvalues of the closed-loop system matrix.

Eigenvalues of the closed-loop active suspension system are presented in Table II. The active suspension system has two pairs of complex conjugate eigenvalues. A system with two pairs of complex conjugate eigenvalues will oscillate in two oscillation modes. Each oscillation mode is characterized by a natural frequency and a damping ratio that can be determined based on the system eigenvalues. Using the theorem 1, the characteristics of both oscillations modes in the active suspension system are calculated, and the results are presented in Table 2.

Theorem 1. A system that has a pair of complex conjugate eigenvalues $\lambda = -a \pm jb$ is going to oscillate with the following frequency and damping ratio:

$$\omega = \sqrt{a^2 + b^2}$$

$$\zeta = \frac{-a}{\sqrt{a^2 + b^2}}$$

where ω is the oscillation frequency, and ζ is the damping ratio [28].

Table 2. Characteristic of the Active System and the Passive Suspension System

Parameter	Mode 1	Mode 2
Active Suspension		
Eigenvalues (λ)	$-10.63 \pm j10.15$	$-17.70 \pm j91.81$
Frequency (ω)	14.7	93.5
Damping ratio (ζ)	0.72	0.19
Passive suspension		
Eigenvalue (λ)	$-1.81 \pm j7.72$	$-23.19 \pm j73.85$
Frequency (ω)	7.93	77.41
Damping ratio (ζ)	0.227	0.3

Performance of the active suspension system is evaluated by comparing it to a passive suspension system. A passive suspension system is similar to an active suspension system except for the active element. The passive suspension system consists of spring and damper. The dynamics of the passive suspension system are derived in the same way as the derivation of active suspension system. The free-body diagram shown in Fig.1 is applied in deriving the passive suspension system by neglecting the active element u . Derivation of the passive suspension system dynamics results in the following state equation:

$$\dot{x} = Ax + Dw \quad (23)$$

where A is the system matrix, and D is the disturbance matrix D , and both are the same as defined in (8).

The characteristic of the passive suspension system is determined by the eigenvalues of passive suspension system. Substituting the passive suspension system parameters given in Table I into (8) and calculating the eigenvalues of A results in the eigenvalues of passive suspension system as listed in Table II. Similar to the active suspension system, the passive suspension system has two pairs of complex conjugate eigenvalues such that it has two oscillation modes. The frequency and damping ratio of each oscillation modes in the passive suspension system are listed in Table 2.

Both suspension systems are simulated through passing a bump road profile. The bump road profile is modeled by the following mathematics function,

$$z_r = \begin{cases} 0, & \text{for } 0 < x_r < 5 \\ \sin(x_r - 5), & \text{for } 5 \leq x_r \leq 5 + \pi \\ 0, & \text{for } x_r > 5 + \pi \end{cases} \quad (24)$$

where x_r is the horizontal road position and z_r is the road elevation. Fig.3 shows the bump road profile.

Simulation results of both active suspension systems and passive suspension systems are presented as follows. The unsprung-mass vertical motions of both active and passive suspension systems are shown in Fig.4. The figure shows that the vertical displacement, vertical velocity, and vertical acceleration of unsprung mass of both active and passive suspension systems are not too different. The unsprung mass is representing the wheel assy. The wheel has direct contact with the road disturbance through the tyre. The tyre is modeled as a spring. The energy of the road disturbance is transmitted to the unsprung mass without any energy dissipation. This results in the vertical motions of unsprung mass follow

generates force to counter the road-disturbance energy. The active element improves dissipating road-disturbance energy. It makes the less road-disturbance energy received by the sprung mass on the active suspension system than the passive suspension system. This is confirmed by the simulation result shown in Fig.5. The sprung-mass vertical motion is stabilized faster using the active suspension system than using the passive suspension system. The sprung mass experiences less oscillation using the active suspension system than the passive suspension system. The less oscillating indicates, the less energy on the system. This implicates that the active suspension system has been dissipated the road-disturbance energy more than the passive suspension system.

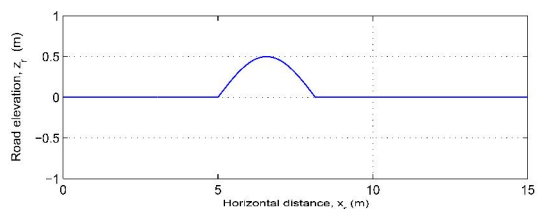


Fig.3. Bump Road Profile.

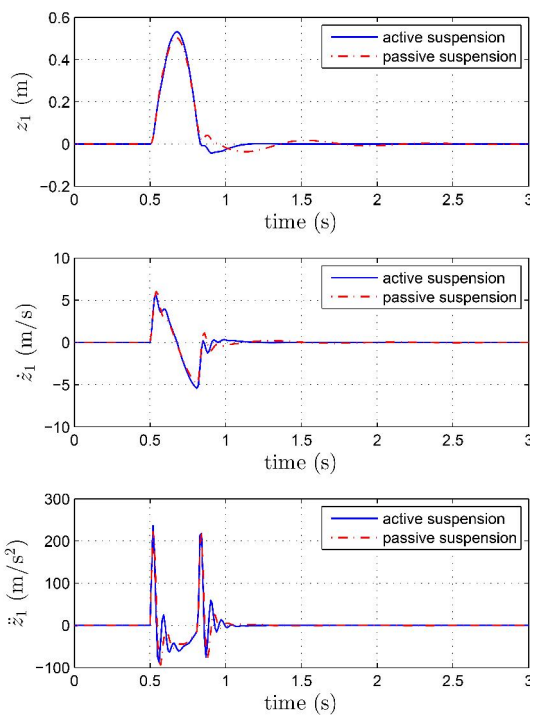


Fig.4. Vertical Movement of The Unsprung Mass.

The road-disturbance energy received by the unsprung mass is continued to be transmitted to the sprung mass through the vehicle suspension. The vehicle suspension consists of spring and damper, and an active element for the active suspension. The damper dissipates the transmitted energy such that the received road-disturbance energy at the sprung mass is less than at the unsprung mass. This is shown by the vertical motions of sprung mass has less amplitude than the unsprung mass motions, as presented in Fig.5. The active element of active suspension system

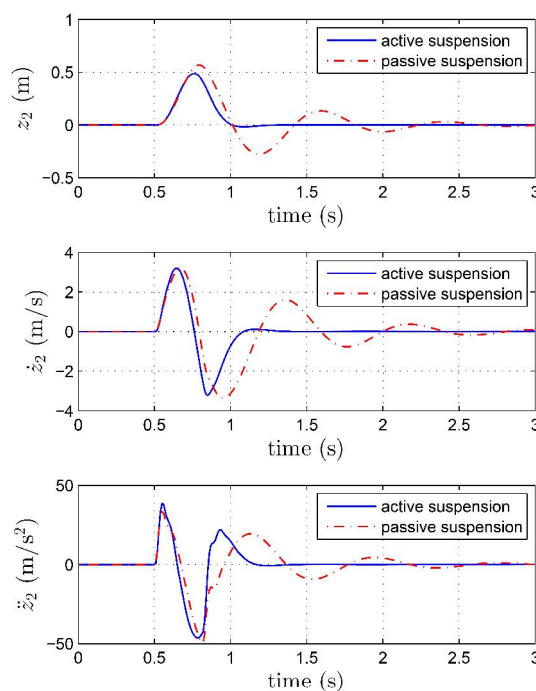


Fig.5. Vertical movement of the sprung mass.

IV. DISCUSSION

Simulations of a quarter-car suspension system passing a bump road profile has been carried out in the previous section. Figure 4 and Fig.5 show the time response of unsprung mass and sprung mass when two identically vehicles but different suspension systems passing on a bump road profile. Both figures provide a qualitative representation of the suspension system's performance. Suspension system performance can be evaluated based on such kind of figures. However, it is quite difficult to appraise the better suspension system performance based on Fig. 4. A quantitative representation is required for better evaluation. Root mean square (RMS) of the unsprung mass and sprung mass motions are commonly used in the qualitative representations of suspension system performance [7]. The less RMS indicates that the suspension system has a better performance.

Definition 1. RMS of a measured state q is defined by the following equation,

$$\bar{q} = \frac{1}{N} \sum_{i=1}^N q_i^2$$

where \bar{q} is the RMS of the measured state, N is the number of sampling data, and q_i is the value of q at the i^{th} sampling.

The RMS calculations are done for the vertical motions of both sprung masses for the cases of using active suspension system and passive suspension system. The calculated RMS of vertical motions includes vertical displacement, vertical velocity, and vertical acceleration. The resulted RMS is normalized to the RMS of using passive suspension systems. This is to show a comparison of the active suspension system performance relative to the passive suspension system performance.

Table 3. RMS of the Unsprung Mass and the Sprung Mass Motions

Vertical motion	Unsprung mass		Sprung mass	
	Active suspension	Passive suspension	Active suspension	Passive suspension
Displacement	106.15	100	49.63	100
Velocity	111.32	100	54.71	100
Acceleration	101.8	100	98.23	100

Table 3 shows the RMS of vertical motions of the unsprung mass and the sprung mass that is calculated based on the vertical motion plotted in Fig. 4 and Fig. 5. For the unsprung mass vertical motion, the active suspension system results in a slightly higher RMS than the passive suspension system. The RMS unsprung mass vertical motion of using active suspension system increased 6.15% on vertical displacement, 11.32% on vertical velocity, and 1.8% on vertical acceleration compared to the one using passive suspension system. However, for the sprung mass vertical motion, the active suspension system results in much lower RMS than the passive suspension system, especially on the vertical displacement and the vertical velocity. The active suspension system shows significant improvement in isolating the sprung mass motion due to the road disturbance than the passive suspension system. The active suspension system decreased the RMS of sprung mass vertical motion significantly that includes 50.37% on vertical displacement, 45.29% on vertical velocity, and 1.77% on vertical acceleration lower than the passive suspension system. The reduction of vertical acceleration is very small compared to the reduction of vertical displacement and the vertical velocity. This is because the matrix Q does not accommodate a weighting factor for minimizing the vertical acceleration.

Although applying the active suspension system result in increasing RMS of unsprung mass motion, but the increment is not too much. The maximum

increment of the RMS was 11.32% on vertical velocity of the unsprung mass. The increasing RMS at the unsprung mass does not decrease the vehicle ride comfort as the passenger seats are located on the sprung mass. The vehicle ride comfort is only determined by the sprung mass motions. Therefore, the simulation results show that the vehicle ride comfort was improved significantly using the active suspension system.

V. CONCLUSION

A study on the active suspension system of ground vehicles has been presented. The suspension system was modeled by a quarter car model. The dynamics of the system were derived and represented in a state-space form. LQR was applied in designing a control law of the active suspension system. The computer simulation was done to evaluate performance of the active suspension system concerning a passive suspension system. The simulation results showed that the active suspension system has much better performance than the passive suspension system. The active suspension system has isolated the vehicle body mass better than the passive suspension system. The active suspension system reduced the vertical motion of sprung mass significantly by reducing 50.37% the vertical displacement, 45.29% the vertical velocity, and 1.77% the vertical acceleration than using the passive suspension system.

The simulation results also showed that the unsprung-mass vertical motion was increased by applying the active suspension. The increment was 6.15% on vertical displacement, 11.32% on vertical velocity, and 1.8% on vertical acceleration. However, the increment of unsprung-mass vertical motion is relatively small compared to the reduction of sprung-mass vertical motion. Moreover, the vehicle ride comfort is determined by the sprung mass motion but not the unsprung mass motion. Therefore, the active suspension system is promising a benefit of better ride comfort than the passive suspension system.

REFERENCES

- [1] J. L. Coyte, D. Stirling, H. Du, and M. Ros, "Seated whole-body vibration analysis, technologies, and modeling: a survey," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 6, pp. 725–739, 2016.
- [2] Y. Mohammadi and S. Ganjefar, "Quarter car active suspension system: Minimum time controller design using singular perturbation method," *International Journal of Control, Automation, and Systems*, vol. 15, no. 6, pp. 2538–2550, 2017.
- [3] H. E. Tseng and D. Hrovat, "State of the art survey: active and semi-active suspension control," *Vehicle System Dynamics*, vol. 53, no. 7, pp. 1034–1062, 2015.
- [4] A. A. Aly and F. A. Salem, "Vehicle suspension systems control: a review," *International Journal of*

- Control, Automation and Systems*, vol. 2, no. 2, pp. 46–54, 2013.
- [5] H. Elahi, A. Israr, M. Z. Khan, and S. Ahmad, “Robust vehicle suspension system by converting active and passive control of a vehicle to semi-active control system analytically,” *Journal of Automation and Control Engineering*, vol. 4, no. 4, 2016.
- [6] M. M. ElMadany and Z. S. Abduljabbar, “Linear quadratic Gaussian control of a quarter-car suspension,” *Vehicle System Dynamics*, vol. 32, no. 6, pp. 479–497, 1999.
- [7] I. Youn, R. Tchamna, S. Lee, N. Uddin, S. Lyu, and M. Tomizuka, “Preview suspension control for a full tracked vehicle,” *International Journal of Automotive Technology*, vol. 15, no. 3, pp. 399–410, 2014.
- [8] S. Bououden, M. Chadli, and H. R. Karimi, “A robust predictive control design for nonlinear active suspension systems,” *Asian Journal of Control*, vol. 18, no. 1, pp. 122–132, 2016.
- [9] S.A. Chen, J.C. Wang, M. Yao, and Y.B. Kim, “Improved optimal sliding mode control for a non-linear vehicle active suspension system,” *Journal of Sound and Vibration*, vol. 395, pp. 1–25, 2017.
- [10] D. Ning, S. Sun, H. Li, H. Du, and W. Li, “Active control of an innovative seat suspension system with acceleration measurement based friction estimation,” *Journal of Sound and Vibration*, vol. 384, pp. 28–44, 2016.
- [11] C. Gohrle, A. Schindler, A. Wagner, and O. Sawodny, “Design and vehicle implementation of preview active suspension controllers,” *IEEE Transactions on Control Systems Technology*, vol. 22, no. 3, pp. 1135–1142, 2014.
- [12] G. Wang, C. Chen, and S. Yu, “Robust non-fragile finite-frequency H_∞ Static output-feedback control for active suspension systems,” *Mechanical Systems and Signal Processing*, vol. 91, pp. 41–56, 2017.
- [13] M. Omar, M. El-Kassaby, and W. Abdelghaffar, “A universal suspension test rig for electro hydraulic active and passive automotive suspension system,” *Alexandria Engineering Journal*, vol. 56, no. 4, pp. 359–370, 2017.
- [14] L. Xiao and Y. Zhu, “Sliding-mode output feedback control for active suspension with nonlinear actuator dynamics,” *Journal of Vibration and Control*, vol. 21, no. 14, pp. 2721–2738, 2015.
- [15] B. Erol and A. Delibasi, “Proportional–integral–derivative type H_∞ controller for quarter car active suspension system,” *Journal of Vibration and Control*, vol. 24, no. 10, pp. 1951–1966, 2018.
- [16] R. S. Prabakar, C. Sujatha, and S. Narayanan, “Response of a half-car model with optimal magneto rheological damper parameters,” *Journal of Vibration and Control*, vol. 22, no. 3, pp. 784–798, 2016.
- [17] M. Assahubulkahfi, Y. M. Sam, A. Maseleno, and M. Huda, “LQR tuning by particle swarm optimization of full car suspension system,” *International Journal of Engineering & Technology (UAE)*, vol. 7, no.2.13, pp. 328–331, 2018.
- [18] Y. Z. Yin, Z. L. Yang, Z. X. Yin, and F. Xu, “Optimal control of LQR for discrete time-varying systems with input delays,” *International Journal of Systems Science*, vol. 49, no. 5, pp. 1021–1031, 2018.
- [19] V. Satyanarayana, B. Sateesh, and N. M. Rao, “Parameters optimization of vehicle suspension system for better ride comfort,” *International Journal of Vehicle Performance*, vol. 4, no. 2, pp. 186–199, 2018.
- [20] S. A. Chen, Y. M. Cai, J. Wang, and M. Yao, “A novel LQG controller of active suspension system for vehicle roll safety,” *International Journal of Control, Automation and Systems*, vol. 16, no. 5, pp. 2203–2213, 2018.
- [21] H. Pang, Y. Chen, J. Chen, and X. Liu, “Design of LQG controller for active suspension without considering road input signals,” *Shock and Vibration*, vol. 2017, 2017.
- [22] Q. Zhu, J. J. Ding, and M. L. Yang, “LQG control based lateral active secondary and primary suspensions of high-speed train for ride quality and hunting stability,” *IET Control Theory & Applications*, vol. 12, no. 10, pp. 1497–1504, 2018.
- [23] C. Gohrle, A. Schindler, A. Wagner, and O. Sawodny, “Road profile estimation and preview control for low-bandwidth active suspension systems,” *IEEE/ ASME Transactions on Mechatronics*, vol. 20, no. 5, pp. 2299–2310, 2015.
- [24] Y. M. Sam, J. H. Osman, and M. R. A. Ghani, “A class of proportional-integral sliding mode control with application to active suspension system,” *Systems & Control Letters*, vol. 51, no. 3–4, pp. 217–223, 2004.
- [25] K. Ogata, *Modern control engineering*. Prentice Hall Upper Saddle River, NJ, 2009.
- [26] P. N. Paraskevopoulos, *Modern control engineering*. CRC Press, 2017.
- [27] A. E. Bryson, *Applied optimal control: optimization, estimation, and control*. Routledge, 2018.
- [28] S. S. Rao, *Mechanical Vibrations 5th Ed*. Pearson, New Jersey, 2011